

# Differential regularization of topologically massive Yang-Mills theory and Chern-Simons theory

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We apply a differential renormalization method to the study of three-dimensional topologically massive Yang-Mills and Chern-Simons theories. The method is especially suitable for such theories as it avoids the need for dimensional continuation of a three-dimensional antisymmetric tensor and the Feynman rules for three-dimensional theories in coordinate space are relatively simple. The calculus involved is still lengthy but not as difficult as other existing methods of calculation. We compute one-loop propagators and vertices and derive the one-loop local effective action for topologically massive Yang-Mills theory. We then consider Chern-Simons field theory as the large mass limit of topologically massive Yang-Mills theory and show that this leads to the famous shift in the parameter  $k$ . Some useful formulas for the calculus of differential renormalization of three-dimensional field theories are given in an Appendix. [S0556-2821(97)07006-9]

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## I. INTRODUCTION

The differential renormalization (DR) method was proposed by Freedman, Johnson, and Latorre [1] to deal with the ultraviolet divergences of quantum field theories a few years ago. Its original idea came from the observation that primitively divergent amplitudes are well defined in coordinate space for noncoincident points, but too singular at short distance to allow a Fourier transform into momentum space. They proposed to renormalize such an amplitude by first writing its singular parts as derivatives of some less singular functions that have well-defined Fourier transformations, then performing Fourier transformations of such functions and discarding the surface terms. This idea is clearly illustrated when one applies it to the one-loop four-point bubble graph of massless  $\phi^4$  theory in four-dimensional space-time. As we know, the amplitude of this graph involves the function  $1/x^4$  that is singular at  $x=0$ , corresponding to an ultraviolet divergence. To realize its differential renormalization we follow Ref. [1] and use the identity

$$\frac{1}{x^4} = -\frac{1}{4} \square \frac{\ln(x^2 M^2)}{x^2} \quad \text{for } x \neq 0. \quad (1)$$

The function  $\ln(x^2 M^2)/x^2$  has a well-defined Fourier transform  $4\pi^2 \ln(p^2/\bar{M}^2)/p^2$ , where  $\bar{M} = M/\gamma$  and  $\gamma$  is Euler's constant. After discarding the surface term we are left with  $-\pi^2 \ln(p^2/\bar{M}^2)$  as the regulated Fourier transform of  $1/x^4$ .

The DR method has been applied to many cases including massless  $\phi^4$  theory up to three-loop order [1], one-loop massive  $\phi^4$  theory [2], supersymmetric Wess-Zumino model up to three loops [3], Yang-Mills theory in background field method up to one-loop [1], QED up to two loops [4] and low dimensional Abelian gauge theories to one-loop [5]. Its rela-

tion with the conventional dimensional regularization in some theories [6,22] and compatibility with unitarity have been investigated [7] and it has been shown to be simpler and more powerful than other regularizations in many cases.

In this paper we shall use the DR method to study the perturbative three dimensional topologically massive Yang-Mills (TMYM) theory and Chern-Simons (CS) theory which, as will be shown, it is especially suited for.

The action of TMYM theory [8], which is obtained by adding to the standard non-Abelian gauge action, the Chern-Simons term, can be written in Euclidean space as

$$S_m = -i \frac{k}{4\pi} \int_x \epsilon^{\mu\nu\rho} \left( \frac{1}{2} A_\mu^a \partial_\nu A_\rho^a + \frac{1}{3!} f^{abc} A_\mu^a A_\nu^b A_\rho^c \right) + \frac{|k|}{16m\pi} \int_x F_{\mu\nu}^a F^{\mu\nu a}, \quad (2)$$

where the integration  $\int_x \equiv \int d^3x$  is over the whole  $R^3$ . The first term, i.e., the Chern-Simons term, exists only in three dimensions. It is easy to see that under a gauge transformation  $U$  the action transforms as

$$S_m \rightarrow S_m - 2\pi i k S_{\text{WZ}},$$

$$S_{\text{WZ}} = \frac{1}{24\pi^2} \int_x \epsilon^{\mu\nu\rho} \text{Tr}(U^{-1} \partial_\mu U U^{-1} \partial_\nu U U^{-1} \partial_\rho U).$$

As it is well known, the Wess-Zumino term  $S_{\text{WZ}}$  takes integer value, so the theory is expected to be gauge invariant at the quantum level when  $k$  takes integer value. At the same

time, an interesting property of  $S_m$  is that the gauge excitations are massive, with mass  $m$ . This property exists only in three dimensions and is not shared by other dimensions.

The perturbative property of TMYM theory was studied in [8,9], where it was pointed out that the computations involved are not trivial and require diligence. In our view, dimensionality plays an important role in defining three-dimensional TMYM theory because much of the topological properties of the theory are derived from the properties of three dimensional antisymmetric tensor  $\epsilon^{\mu\nu\rho}$ . A calculation without using dimensional continuation is therefore called for. We are furthermore motivated to study the theory with DR by the fact that in three dimensions, propagators of TMYM in coordinate space have analytic forms that are particularly suited for the application of this method.

During the past several years a number of studies of perturbative Chern-Simons theory have been carried out using a variety of regularization schemes including: higher covariant derivative (HCD) combined with generalized Pauli-Villars regularization [10]; HCD combined with dimensional regularization [11,12]; operator regularization [13];  $\eta$  function regularization [14]; geometric regularization [15]; and Feynman propagator regularization [16]. Especially recently an understanding on the perturbative behavior of CS from supersymmetric Yang-Mills-Chern-Simons theory has appeared [17] and a more strict mathematical treatment from the geometric viewpoint has been discussed in Ref. [18]. From these studies there emerges the so-called  $k$ -shift problem in three-dimensional CS, which is concerned with whether quantum correction change the value of the parameter  $k$ . It appears that whether the value of  $k$  shifts or not depends on the regularization scheme [19,20]—some of the calculations in these studies showed the  $k$  shift while others did not. In Ref. [19], an analysis shows a family of shift can be generated, which depends on the parity property of the regulator.

As we know, Chern-Simons action is just the first term in Eq. (2). Obviously we can consider TMYM theory as a partially (high covariant derivative) regulated version of CS or, alternatively, CS as the large mass limit ( $m \rightarrow \infty$ ) of TMYM [11]. So a calculation of the perturbative property of three-dimensional TMYM yields a study of the  $k$ -shift problem of three-dimensional CS as a by-product. Our result confirms the existence of  $k$  shift and coincides with the case of scalar regulators of Ref. [19].

This paper is organized as follows. In Sec. II we present the Feynman rules of TMYM in coordinate space. Section III is devoted to explicit calculations of one-loop amplitudes needed for the computation of one-loop local effective action, where we have obtained ghost self-energy, vacuum polarization tensor and gauge boson-ghost-ghost vertex. In Sec. IV Slavnov-Taylor identity is explicitly derived and used in combination with results from Sec. III to determine the one-loop local effective action. In Sec. V, as an example demonstrating the usefulness of formulas given in Appendix, we give a result for the self-energy of gauge field in three-dimensional QED. In Sec. VI, we discuss and summarize the results. Some formulas utilized in the calculation are given in the Appendix. These formulas should also be useful for DR calculations of other low-dimensional field theories.

## II. FEYNMAN RULES IN COORDINATE SPACE

By defining  $g^2 = 4\pi/|k|$  and rescaling  $A \rightarrow A/g$ , we rewrite TMYM action (1) as

$$S_m = -i \operatorname{sgn}(k) \int_x \epsilon^{\mu\nu\rho} \left( \frac{1}{2} A_\mu^a \partial_\nu A_\rho^a + \frac{1}{3!} g f^{abc} A_\mu^a A_\nu^b A_\rho^c \right) + \frac{1}{4m} \int_x F_{\mu\nu}^a F^{\mu\nu a}, \quad (3)$$

whose corresponding Becchi-Rouet-Stora-Tyutin- (BRST-) invariant action in the Landau gauge is

$$S[A, c, \bar{c}, B, m] = S_m + \int_x [\partial_\mu \bar{c}^a D^\mu c^a + B^a \partial_\mu A^{\mu a}]. \quad (4)$$

The BRST transformation of the fields are

$$\begin{aligned} \delta A_\mu^a &= D_\mu c^a, & \delta \bar{c}^a &= B^a, \\ \delta c^a &= -\frac{1}{2} g f^{abc} c^b c^c, & \delta B^a &= 0. \end{aligned} \quad (5)$$

Here we choose the Landau gauge because of its good infrared behavior [9]. For a pure Chern-Simons field theory, the Landau vector supersymmetry [20,23,24]

$$\begin{aligned} v_\mu A_\nu^a &= i \operatorname{sgn}(k) \epsilon_{\nu\mu\rho} \partial^\rho c^a, & v_\mu c^a &= 0, \\ v_\mu \bar{c}^a &= A_\mu^a, & v_\mu B^a &= -D_\mu c^a, \end{aligned} \quad (6)$$

which only exists in the Landau gauge, plays a crucial role in the cancellation of the infrared divergence. Although the inclusion of Yang-Mills term in TMYM theory breaks this symmetry, it does not ruin the cancellation of the infrared singularity.

The generating functional can be formally written as

$$Z[J, \eta, \bar{\eta}, M] = \int \mathcal{D}A \mathcal{D}B \mathcal{D}c \mathcal{D}\bar{c} \exp \left( -S - \int [J_\mu^a A_\mu^a + \bar{\eta}^a c^a + \bar{c}^a \eta^a + B^a M^a] \right). \quad (7)$$

Differential regularization works in coordinate space, so we need the Feynman rules in coordinate space. Defining

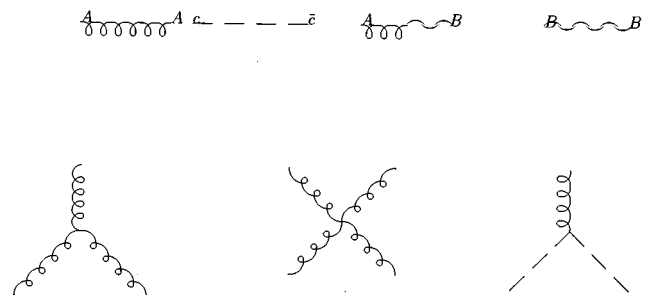


FIG. 1. Feynman rules.

$$\begin{aligned}
G_{\mu\nu}^{ab}(x-y) &= \langle 0|T[A_\mu^a(x)A_\nu^b(y)]|0\rangle, \\
\Lambda_\mu^{ab}(x-y) &= \langle 0|T[A_\mu^a(x)B^b(y)]|0\rangle, \\
\Lambda^{ab}(x-y) &= \langle 0|T[B^a(x)B^b(y)]|0\rangle, \\
S^{ab}(x-y) &= \langle 0|T[c^a(x)\bar{c}^b(y)]|0\rangle, \\
\langle 0|T[A_\mu^a(x)A_\nu^b(y)A_\rho^c(z)]|0\rangle &= \int_{x'} \int_{y'} \int_{z'} G_{\mu\mu'}^{aa'}(x-x') G_{\nu\nu'}^{bb'}(y-y') G_{\rho\rho'}^{cc'}(z-z') \Gamma_{(3)\mu'\nu'\rho'}^{a'b'c'}(x',y',z'), \\
\langle 0|T[A_\mu^a(x)A_\nu^b(y)A_\rho^c(z)A_\sigma^d(w)]|0\rangle &= \int_{x'} \int_{y'} \int_{z'} G_{\mu\mu'}^{aa'}(x-x') G_{\nu\nu'}^{bb'}(y-y') \\
&\quad \times G_{\rho\rho'}^{cc'}(z-z') G_{\sigma\sigma'}^{dd'}(w-w') \Gamma_{(4)\mu'\nu'\rho'\sigma'}^{a'b'c'd'}(x',y',z',w'), \\
\langle 0|T[c^a(x)\bar{c}^c(z)A_\mu^b(y)]|0\rangle &= \int_{x'} \int_{y'} \int_{z'} S^{aa'}(x-x') G_{\mu\mu'}^{bb'}(y-y') S^{c'c}(z'-z) \times \Lambda_{\mu'}^{a'b'c'}(x',y',z'), \tag{8}
\end{aligned}$$

we obtain Feynman rules as (Fig. 1)

$$\begin{aligned}
G_{\mu\nu}^{(0)ab}(x-y) &\equiv \delta^{ab} D_{\mu\nu}(x-y) \\
&= -\delta^{ab} \left[ i \operatorname{sgn}(k) \epsilon_{\mu\nu\rho} \partial_x^\rho + \frac{1}{m} (\delta_{\mu\nu} \nabla^{2x} - \partial_\mu^x \partial_\nu^x) \right] \frac{(1 - e^{-m|x-y|})}{4\pi|x-y|}, \\
\Lambda_\mu^{(0)ab}(x-y) &\equiv \delta^{ab} \Lambda_\mu(x-y) = -\delta^{ab} \partial_\mu^x \frac{1}{4\pi|x-y|}, \\
\Lambda^{(0)ab}(x-y) &= 0, \\
S^{(0)ab}(x-y) &\equiv \delta^{ab} S(x-y) = \delta^{ab} \frac{1}{4\pi|x-y|}, \\
\Gamma_{(3)\mu\nu\rho}^{(0)abc}(x,y,z) &= g f^{abc} \left\{ i \operatorname{sgn}(k) \epsilon_{\mu\nu\rho} - \frac{1}{m} [(\partial_\mu^y - \partial_\mu^z) \delta_{\nu\rho} + (\partial_\nu^z - \partial_\nu^x) \delta_{\rho\mu} + (\partial_\rho^x - \partial_\rho^y) \delta_{\mu\nu}] \right\} \\
&\quad \times \int_u \delta^{(3)}(x-u) \delta^{(3)}(y-u) \delta^{(3)}(z-u), \\
\Gamma_{(4)\mu\nu\rho\sigma}^{(0)abcd}(x,y,z,w) &= \frac{g^2}{m} [f^{eab} f^{ecd} (\delta_{\nu\rho} \delta_{\sigma\mu} - \delta_{\nu\sigma} \delta_{\mu\rho}) + f^{eac} f^{edb} (\delta_{\rho\sigma} \delta_{\nu\mu} - \delta_{\nu\rho} \delta_{\mu\sigma}) + f^{ead} f^{ebc} (\delta_{\sigma\nu} \delta_{\rho\mu} - \delta_{\sigma\rho} \delta_{\mu\nu})] \\
&\quad \times \int_u \delta^{(3)}(x-u) \delta^{(3)}(y-u) \delta^{(3)}(z-u) \delta^{(3)}(w-u), \\
\Lambda_\mu^{(0)abc}(x,y,z) &= g f^{abc} \partial_\mu^x \int_u \delta^{(3)}(x-u) \delta(y-u) \delta(z-u), \tag{9}
\end{aligned}$$

where the superscript “(0)” denotes free propagators or bare vertices. We can see that the propagators given above are much simpler in comparison with their counterparts in four-dimensional massive theories, which are Bessel functions [21].

### III. ONE-LOOP AMPLITUDES

Now we use DR to carry out the one-loop renormalization of TMYM theory. Naive power counting suggests that

some of the one-loop diagrams should be ultraviolet divergent. But as we will show, in some sense, TMYM is essentially a finite theory [9]. Purely for the purpose of making this finiteness manifest (DR does not require it), we introduce a short distance cutoff by excluding a small ball  $B_\epsilon$  of radius  $\epsilon$  about the origin as in [1,7]. Denote the region  $R^3 - B_\epsilon$  by  $R_\epsilon^3$ .

Let us analyze the one-loop ghost self-energy first. Its Feynman diagram is shown in Fig. 2. The Fourier transformation of its amplitude is

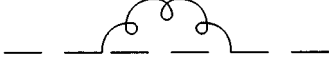


FIG. 2. Ghost self-energy.

$$-g^2 C_V \delta^{ab} \int_{R_\epsilon^3} \partial_\mu^x (e^{-ip \cdot x}) D_{\mu\nu}(x) \partial_\nu^x S(x). \quad (10)$$

Here we would like to emphasize that we need to be careful about the positions of the partial differential operators in the Feynman rules of Eq. (9). By using Eq. (9) and the formulas in the Appendix we have

$$S^{(1)ab}(p) = \delta^{ab} \frac{g^2 C_V}{8\pi^2 m} \int_{R_\epsilon^3} e^{-ip \cdot x} i p_\mu x_\mu \left[ \frac{1 - e^{-mr}}{r^6} - \frac{m e^{-mr}}{r^5} \right]. \quad (11)$$

Writing singular functions at  $r=0$  in the above integrand as derivatives of less singular functions, we get

$$S^{(1)ab}(p) = -\delta^{ab} \frac{g^2 C_V}{8\pi^2 m} \int_{R_\epsilon^3} e^{-ip \cdot x} i p_\mu \partial_\mu \left[ \nabla^2 \frac{1 - e^{-mr}}{8r^2} + \frac{m^2 e^{-mr}}{4r^2} - \frac{m^3 e^{-mr}}{6r} - \frac{m^4 \text{Ei}(-mr)}{8} \right], \quad (12)$$

where  $\text{Ei}(x)$  is the exponential integral function:

$$\text{Ei}(-mr) = \int_{-\infty}^{-mr} dt \frac{e^t}{t}. \quad (13)$$

Particular attention should be paid to differential operators in Eq. (12) when we perform the Fourier transformation. For example,

$$\begin{aligned} & \int_{R_\epsilon^3} e^{-ip \cdot x} \nabla^2 \frac{1 - e^{-mr}}{8r^2} \\ &= \int_{R_\epsilon^3} \left[ \nabla^2 \left( e^{-ip \cdot x} \frac{1 - e^{-mr}}{8r^2} \right) - 2 \partial_\mu e^{-ip \cdot x} \right. \\ & \quad \left. \times \partial_\mu \frac{1 - e^{-mr}}{8r^2} + p^2 e^{-ip \cdot x} \frac{1 - e^{-mr}}{8r^2} \right] \\ &= \frac{m\pi}{2} + p^2 \int_{R_\epsilon^3} e^{-ip \cdot x} \frac{1 - e^{-mr}}{8r^2}, \end{aligned} \quad (14)$$

where the first term on the right-hand side is a surface term from the cutoff ball  $B_\epsilon$ . Finally, using the formulas in the Appendix, we obtain the one-loop ghost self-energy

$$S^{(1)ab}(p) = -\delta^{ab} \frac{g^2 C_V}{16\pi^2} p^2 \left[ \frac{\pi p}{2m} + \frac{m^2}{p^2} - 1 - \frac{m}{p} \times \left( \frac{p}{m} - \frac{m}{p} \right)^2 \arctan \frac{p}{m} \right], \quad (15)$$

where  $p \equiv |p|$ .

The one-loop gluon vacuum polarization part can be computed in a similar way. The proper gluon self-energy is determined by gauge symmetry to have the form:

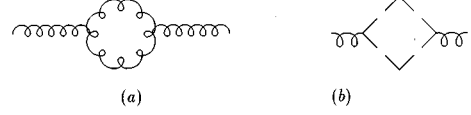


FIG. 3. Vacuum polarization tensor.

$$\begin{aligned} \Pi_{\mu\nu}^{ab}(p) &= \delta^{ab} [\Pi_{o\mu\nu}(p) + \Pi_{e\mu\nu}(p)] \\ &= \delta^{ab} \left[ \text{sgn}(p) \epsilon_{\mu\nu\rho} p_\rho \Pi_o(p^2) - \frac{1}{m} (\delta_{\mu\nu} p^2 - p_\mu p_\nu) \Pi_e(p^2) \right], \end{aligned} \quad (16)$$

where the subscripts ‘‘o’’ and ‘‘e’’ denote parity-odd and parity-even, respectively. The single ghost-loop contribution to the vacuum polarization tensor [Fig. 3(b)] is

$$-\delta^{ab} \frac{g^2 C_V}{16\pi^2 m} \int_{R_\epsilon^3} e^{-ip \cdot x} \partial_\mu \frac{1}{r} \partial_\nu \frac{1}{r}. \quad (17)$$

Combining it with the contribution from the singular gluon-loop, we have

$$\begin{aligned} \Pi_{o\mu\nu}(p) &= i \text{sgn}(k) \frac{g^2 C_V}{16\pi^2} \int_{R_\epsilon^3} e^{-ip \cdot x} \epsilon_{\mu\nu\rho} \partial^\rho \left[ \frac{9}{m^3 r^6} (1 - e^{-mr})^2 \right. \\ & \quad - \frac{18}{m^2 r^5} (1 - e^{-mr}) e^{-mr} + \frac{1}{m r^4} \left( -1 - \frac{13}{2} e^{-mr} \right. \\ & \quad \left. \left. + \frac{33}{2} e^{-2mr} \right) - \frac{1}{2r^3} e^{-mr} + \frac{9}{r^3} e^{-2mr} + \frac{m}{4r^2} e^{-mr} \right. \\ & \quad \left. \left. - \frac{m^2}{4r} e^{-mr} - \frac{m^3}{4} \text{Ei}(-mr) \right] \\ &= i \text{sgn}(k) \frac{g^2 C_V}{16\pi^2} \int_{R_\epsilon^3} e^{-ip \cdot x} \epsilon_{\mu\nu\rho} \partial^\rho \\ & \quad \times \left[ \frac{3}{8m^3} (\nabla^2)^2 \frac{(1 - e^{-mr})^2}{r^2} + \nabla^2 \left( -\frac{1}{2} + \frac{5}{4} e^{-mr} \right. \right. \\ & \quad \left. \left. - \frac{3}{4} \frac{e^{-2mr}}{m r^2} \right) + \frac{m}{r^2} \left( -\frac{1}{4} e^{-mr} - 3e^{-2mr} \right) \right. \\ & \quad \left. - \frac{m^2}{4r} e^{-mr} - \frac{m^3}{4} \text{Ei}(-mr) \right]. \end{aligned} \quad (18)$$

Again after performing a Fourier transformation we obtain

$$\begin{aligned} \Pi_o(p^2) &= \frac{g^2 C_V}{16\pi^2} \left[ \left( 3 \frac{p^3}{m^3} + 5 \frac{p}{m} - \frac{m}{p} - \frac{m^3}{p^3} \right) \arctan \frac{p}{m} \right. \\ & \quad \left. + \left( -\frac{3}{2} \frac{p^3}{m^3} - 3 \frac{p}{m} + 12 \frac{m}{p} \right) \arctan \frac{p}{2m} - \frac{3}{4} \pi \frac{p^3}{m^3} \right. \\ & \quad \left. - \pi \frac{p}{m} + \frac{m^2}{p^2} + 2 \right]. \end{aligned} \quad (19)$$

The calculation of one gluon-loop contribution to  $\Pi_e(p^2)$  is similar but tedious. The result is

$$\begin{aligned}
 \Pi_\epsilon(p^2) &= \int_{R_\epsilon^3} e^{-ip \cdot x} \frac{g^2}{16\pi^2} C_V \left[ \frac{3}{2m^4 r^6} (1 - e^{-mr})^2 - \frac{3}{m^3 r^5} e^{-mr} (1 - e^{-mr}) + \frac{1}{m^2 r^4} \left( \frac{1}{8} - \frac{11}{4} e^{-mr} + \frac{33}{8} e^{-2mr} \right) \right. \\
 &\quad - \frac{1}{mr^3} \left( \frac{7}{4} e^{-mr} + \frac{17}{4} \right) e^{-2mr} + \frac{1}{r^2} \left( \frac{7}{8} e^{-mr} + \frac{1}{4} e^{-2mr} \right) - \frac{1}{r} \left( \frac{m}{2} e^{-2mr} - \frac{7m}{8} e^{-mr} \right) \\
 &\quad \left. - m^2 \text{Ei}(-2mr) - \frac{7m^2}{8} \text{Ei}(-mr) \right] \\
 &= \int_{R_\epsilon^3} e^{-ip \cdot x} \frac{g^2}{16\pi^2} C_V \left\{ \nabla^4 \left[ \frac{1}{16} \frac{1}{m^4 r^2} (1 - e^{-mr})^2 \right] + \nabla^2 \left[ \frac{1}{m^2 r^2} \left( \frac{1}{16} - \frac{5}{8} e^{-mr} + \frac{9}{16} e^{-2mr} \right) \right] + \frac{1}{r^2} \left( \frac{13}{8} e^{-mr} - 3e^{-2mr} \right) \right. \\
 &\quad \left. - \frac{m}{2r} e^{-2mr} - \frac{7m}{8r} e^{-mr} - m^2 \text{Ei}(-2mr) - \frac{7m^2}{8} \text{Ei}(-mr) \right\} \\
 &= -\frac{g^2 C_V}{32\pi} \left[ \left( -8 \frac{m^3}{p^3} + 24 \frac{m}{p} + \frac{9}{2} \frac{p}{m} - \frac{1}{2} \frac{p^3}{m^3} \right) \arctan \frac{p}{2m} + \left( -7 \frac{m^3}{p^3} - 13 \frac{m}{p} - 5 \frac{p}{m} + \frac{p^3}{m^3} \right) \arctan \frac{p}{m} \right. \\
 &\quad \left. + 11 \frac{m^2}{p^2} + \frac{\pi p}{4m} - \frac{\pi p^3}{4m^3} + 5 \right]. \tag{20}
 \end{aligned}$$

The next step is to construct the local part of the one-loop effective action and to demonstrate renormalization explicitly. From general principles [25] we know that this construction requires at least one one-loop, three-point Green function. Here we choose the one-loop vertex  $A c \bar{c}$ , whose Feynman diagrams is shown in Fig. 4.

The amplitudes, which we know are divergenceless from dimensional analysis, can be written from Fig. 4 as

$$\begin{aligned}
 V_\mu^{abc}(p, q, r) &= \frac{1}{2} g^3 C_V f^{abc} \int_x \int_y e^{-i(p \cdot x + q \cdot y)} [V_\mu^{(a)}(x, y) \\
 &\quad + V_\mu^{(b)}(x, y)], \tag{21}
 \end{aligned}$$

where  $p + q + r = 0$ . The contribution from Fig. 4(a) is

$$\begin{aligned}
 V_\mu^{(a)}(x, y) &= i q_\nu \partial_\mu^x S(x - y) \partial_\rho^x S(x) D_{\nu\rho}(y) \\
 &= -i q_\nu \frac{1}{(4\pi)^3} \partial_\mu^x \frac{1}{|x - y|} \partial_\rho^x \left[ \frac{1}{x} (1 - e^{-mx}) \right] \\
 &\quad \times \left[ -i \text{sgn}(k) \epsilon_{\nu\rho\alpha} \partial_\alpha^y - \frac{1}{m} \delta_{\nu\rho} \nabla^2 y + \frac{1}{m} \partial_\nu^y \partial_\rho^y \right] \frac{1}{y} \\
 &\quad \times (1 - e^{-my}). \tag{22}
 \end{aligned}$$

For our purpose, that is, to construct the local part of the effective action, only the zero-momentum limit of this amplitude is needed,

$$V_\mu^{(a)abc}(p, q, r) = -g^3 \frac{C_V f^{abc}}{2} \frac{17}{36} \frac{1}{4\pi} i q_\mu + \dots \tag{23}$$

The amplitude from Fig. 4(b) can be reduced to

$$\begin{aligned}
 V_\mu^{(b)abc}(p, q, r) &= \frac{C_V f^{abc}}{2} \int_x \int_y e^{i(q \cdot y + r \cdot x)} \left[ -i \text{sgn}(k) \epsilon_{\mu\nu\rho} q_\lambda \partial_\sigma^x S(x - y) \right. \\
 &\quad \times D_{\nu\lambda}(y) D_{\rho\sigma}(x) - \frac{1}{m} p_\rho q_\lambda \partial_\sigma^x S(x - y) D_{\mu\lambda}(y) D_{\rho\lambda}(x) \\
 &\quad + \frac{1}{m} i q_\lambda \partial_\sigma^x S(x - y) \partial_\mu^y D_{\rho\lambda}(y) D_{\rho\sigma}(x) \\
 &\quad - \frac{1}{m} i q_\lambda \partial_\sigma^x S(x - y) \partial_\rho^y D_{\mu\lambda}(y) D_{\rho\sigma}(x) \\
 &\quad + \frac{1}{m} i q_\lambda \partial_\sigma^x S(x - y) D_{\nu\lambda}(y) \partial_\mu^x D_{\nu\sigma}(x) \\
 &\quad \left. - \frac{1}{m} i q_\lambda \partial_\sigma^x S(x - y) D_{\nu\lambda}(y) \partial_\nu^x D_{\mu\sigma}(x) \right. \\
 &\quad \left. + \frac{1}{m} p_\nu q_\lambda \partial_\sigma^x S(x - y) D_{\nu\lambda}(y) D_{\mu\sigma}(x) \right], \tag{24}
 \end{aligned}$$

which, after a similar analysis and a lengthy calculation, yields the zero-momentum limit

$$V_\mu^{(b)abc}(p, q, r) = \frac{C_V f^{abc}}{2} \frac{17}{36} \frac{1}{4\pi} i q_\mu + \dots \tag{25}$$

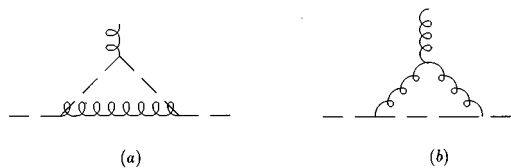


FIG. 4. One-loop ghost-gluon vertex.

From Eqs. (21), (23), and (25), we conclude that one-loop  $Ac\bar{c}$  vertex takes the form

$$V_{\mu}^{abc} = 0 + \dots \quad (26)$$

This means precisely that  $\tilde{Z}(0) = 1$  to one-loop order,  $\tilde{Z}(0)$  denotes the  $Ac\bar{c}$  vertex renormalization constant defined at  $p^2 = q^2 = r^2 = 0$ . In fact, this is the correct result to any order in perturbation expansion for a gauge theory in the Landau gauge.

#### IV. ONE-LOOP LOCAL EFFECTIVE ACTION

Having computed the vacuum polarization tensor, the ghost self-energy, and the  $Ac\bar{c}$  vertex, we are now in a position to derive the local effective action. Our method is the same as that used in [11] for Chern-Simons theory.

We define the generating functional  $Z[J, \eta, \bar{\eta}, M, K, L]$  with the external fields  $K_{\mu}^a$  and  $L^a$ , respectively, coupled to nonlinear BRST transformation products  $D_{\mu}c^a$  and  $-gf^{abc}c^b c^c/2$  as

$$\begin{aligned} Z[J, \eta, \bar{\eta}, M, K, L] = & \int \mathcal{D}X \exp \left\{ - \left[ S + \int_x \left( J_{\mu}^a A^{\mu a} + \bar{\eta}^a c^a \right. \right. \right. \\ & + \bar{c}^a \eta^a + B^a M^a + K_{\mu}^a D^{\mu} c^a \\ & \left. \left. \left. + L^a \left( -\frac{1}{2} g f^{abc} c^b c^c \right) \right) \right] \right\}, \quad (27) \end{aligned}$$

where  $X = (A_{\mu}, B, c, \bar{c})$ . The Slavnov-Taylor identity arising from the BRST transformation in Eq. (5) is

$$\int_x \left[ J_{\mu}^a \frac{\delta}{\delta K_{\mu}^a} - \bar{\eta}^a \frac{\delta}{\delta L^a} + \eta^a \frac{\delta}{\delta M^a} \right] Z = 0. \quad (28)$$

In addition, the invariance of  $Z[J, \eta, \bar{\eta}, M, K, L]$  under the translations  $B^a(x) \rightarrow B^a(x) + \lambda^a(x)$ ,  $\bar{c}^a(x) \rightarrow \bar{c}^a(x) + \omega^a(x)$  leads, respectively, to the  $B$  field and anti-ghost field equations:

$$\left[ \partial_{\mu} \frac{\delta}{\delta J_{\mu}^a} - M^a \right] Z = 0, \quad (29)$$

$$\left[ \partial_{\mu} \frac{\delta}{\delta K_{\mu}^a} - \eta^a \right] Z = 0. \quad (30)$$

Defining the generating functional for the connected Green function  $W$  and that for the one-particle-irreducible Green function  $\Gamma$  (i.e., the quantum effective action) as,

$$W[J, \eta, \bar{\eta}, M, K, L] = - \ln Z[J, \eta, \bar{\eta}, M, K, L],$$

$$\begin{aligned} \Gamma[A_{\mu}^a, B^a, c^a, \bar{c}^a, K_{\mu}^a, L^a] = & W[A_{\mu}^a, B^a, c^a, \bar{c}^a, K_{\mu}^a, L^a] \\ & - (A^{\mu a} J_{\mu}^a + B^a M^a + \bar{\eta}^a c^a + \bar{c}^a \eta^a), \quad (31) \end{aligned}$$

we obtain the actions of the Slavnov-Taylor identity, the  $B$  field and the anti-ghost field equations on  $\Gamma$ :

$$\partial_{\mu} A^{\mu a} + \frac{\delta \Gamma}{\delta B^a} = 0, \quad (32)$$

$$\partial_{\mu} \frac{\delta \Gamma}{\delta K_{\mu}^a} - \frac{\delta \Gamma}{\delta \bar{c}^a} = 0, \quad (33)$$

$$\int_x \left[ \frac{\delta \Gamma}{\delta A^{\mu a}} \frac{\delta \Gamma}{\delta K_{\mu}^a} - \frac{\delta \Gamma}{\delta c^a} \frac{\delta \Gamma}{\delta L^a} \right] = 0. \quad (34)$$

By a redefinition of  $\Gamma$ ,

$$\bar{\Gamma} = \Gamma + \int_x B^a \partial^{\mu} A_{\mu}^a, \quad (35)$$

these equations become

$$\frac{\delta \bar{\Gamma}}{\delta B^a} = 0, \quad \partial_{\mu} \frac{\delta \bar{\Gamma}}{\delta K_{\mu}^a} - \frac{\delta \bar{\Gamma}}{\delta \bar{c}^a} = 0, \quad (36)$$

$$\int_x \left[ \frac{\delta \bar{\Gamma}}{\delta A^{\mu a}} \frac{\delta \bar{\Gamma}}{\delta K_{\mu}^a} - \frac{\delta \bar{\Gamma}}{\delta c^a} \frac{\delta \bar{\Gamma}}{\delta L^a} \right] = 0. \quad (37)$$

The first relation in Eq. (36) means that the redefined action  $\bar{\Gamma}$  is independent of  $B^a$ , and the second relation implies that  $K_{\mu}^a$  and  $\bar{c}^a$  always appear in  $\bar{\Gamma}$  through the combination

$$G_{\mu}^a(x) = K_{\mu}^a - \partial_{\mu} \bar{c}^a. \quad (38)$$

Now we introduce the loop-wise expansion for  $\bar{\Gamma}$

$$\bar{\Gamma} = \sum_{n=0}^{\infty} \hbar^n \bar{\Gamma}^{(n)}, \quad (39)$$

where  $\bar{\Gamma}^{(0)}$  is the classical effective action without the gauge-fixing term  $\int_x B^a \partial^{\mu} A_{\mu}^a$

$$\begin{aligned} \bar{\Gamma}^{(0)} = & -i \operatorname{sgn}(k) \int_x \epsilon^{\mu\nu\rho} \left( \frac{1}{2} A_{\mu}^a \partial_{\nu} A_{\rho}^a + \frac{1}{3!} g f^{abc} A_{\mu}^a A_{\nu}^b A_{\rho}^c \right) \\ & + \frac{1}{4m} \int_x F_{\mu\nu}^a F^{\mu\nu a} \\ & + \int_x \left[ G_{\mu}^a D^{\mu} c^a + L^a \left( -\frac{1}{2} g f^{abc} c^b c^c \right) \right]. \quad (40) \end{aligned}$$

Substituting this expansion into Eq. (37) and comparing the coefficients of the  $\hbar^0$  and  $\hbar^1$  terms lead to

$$\int_x \left[ \frac{\delta \Gamma^{(0)}}{\delta A^{\mu a}} \frac{\delta \Gamma^{(0)}}{\delta G_{\mu}^a} - \frac{\delta \Gamma^{(0)}}{\delta c^a} \frac{\delta \Gamma^{(0)}}{\delta L^a} \right] = 0 \quad (41)$$

and

$$\Delta \Gamma^{(1)} = 0, \quad (42)$$

where we have used the relation

$$\frac{\delta}{\delta G_\mu^a} = \frac{\delta}{\delta K_\mu^a}. \quad (43)$$

is the quantum analogue of the classical BRST operator and is nilpotent

$$\Delta^2 = 0. \quad (45)$$

$\Delta$ , the linear Slavnov-Taylor operator

$$\Delta = \int_x \left[ \frac{\delta \bar{\Gamma}^{(0)}}{\delta A^{\mu a}} \frac{\delta}{\delta G_\mu^a} + \frac{\delta}{\delta A^{\mu a}} \frac{\delta \bar{\Gamma}^{(0)}}{\delta G_\mu^a} - \frac{\delta \bar{\Gamma}^{(0)}}{\delta c^a} \frac{\delta}{\delta L^a} - \frac{\delta}{\delta c^a} \frac{\delta \bar{\Gamma}^{(0)}}{\delta L^a} \right], \quad (44)$$

Now we follow the method of [11,20] to find the solution to Eq. (42). From the requirement of zero ghost-number and mass dimension 3 we determine the general form of one-loop effective action to be that<sup>1</sup>

$$\begin{aligned} \Gamma^{(1)} = & \alpha_1 \left[ -i \operatorname{sgn}(k) \int_x \epsilon^{\mu\nu\rho} \left( \frac{1}{2} A_\mu^a \partial_\nu A_\rho^a + \frac{1}{3!} g f^{abc} A_\mu^a A_\nu^b A_\rho^c \right) \right] \\ & + \alpha_2 \frac{1}{4m} \int_x F_{\mu\nu}^a F^{\mu\nu a} + \Delta \int_x [\beta_1 G^{\mu a} A_\mu^a + \beta_2 L^a c^a] \\ = & -i \operatorname{sgn}(k) (\alpha_1 + 2\beta_1) \int_x \frac{1}{2} \epsilon^{\mu\nu\rho} A_\mu^a \partial_\nu A_\rho^a - i \operatorname{sgn}(k) (\alpha_1 + 3\beta_1) \int_x \frac{1}{3!} \epsilon^{\mu\nu\rho} f^{abc} A_\mu^a A_\nu^b A_\rho^c + (\alpha_2 + 2\beta_1) \frac{1}{4m} \\ & \times \int_x (\partial^\mu A^{\nu a} - \partial^\nu A^{\mu a}) (\partial_\mu A_\nu^a - \partial_\nu A_\mu^a) + (\alpha_2 + 3\beta_1) \frac{1}{2m} \int_x g f^{abc} (\partial_\mu A_\nu^a - \partial_\nu A_\mu^a) A^{\mu b} A^{\nu c} + (\alpha_2 + 4\beta_1) \frac{1}{4m} \\ & \times \int_x g^2 f^{eab} f^{ecd} A_\mu^a A_\nu^b A^{\mu c} A^{\nu d} + \beta_1 \int_x G_\mu^a \partial^\mu c^a + \beta_2 \int_x G_\mu^a D^\mu c^a - \beta_2 \int_x \frac{1}{2} g f^{abc} c^b c^c, \end{aligned} \quad (46)$$

where  $\alpha_i$  and  $\beta_i$  are constant coefficients. In comparison with CS [11,20], we find that the formal large- $m$  limit of above effective action has the same form as that in Refs. [11,20], i.e., the difference lies only in the mass dependent terms. By using the results given in the last section and choosing the renormalization point at  $|p|=0$ , we can determine the values of the parameters as follows:

$$\alpha_1 = \frac{g^2 C_V}{4\pi}, \quad \alpha_2 = -\frac{g^2 C_V}{32\pi}, \quad \beta_1 = -\frac{g^2 C_V}{16\pi}, \quad \beta_2 = 0. \quad (47)$$

Thus, up to one-loop the explicit local effective action is

$$\begin{aligned} \Gamma_{\text{local}} = & \Gamma^{(0)} + \Gamma_{\text{local}}^{(1)} \\ = & \left( 1 + \frac{1}{4\pi} g^2 C_V \right) [-i \operatorname{sgn}(k)] \int_x \epsilon^{\mu\nu\rho} \left( \frac{1}{2} A_\mu^a \partial_\nu A_\rho^a + \frac{1}{3!} A_\mu^a A_\nu^b A_\rho^c \right) + \frac{1}{4m} \left( 1 - \frac{1}{32\pi} g^2 C_V \right) \\ & \times \int_x F^{\mu\nu a} F_{\mu\nu}^a - \Delta \left( \frac{1}{16\pi} g^2 C_V \int_x G_\mu^a A_\mu^a + \int_x L^a c^a \right) + \int_x B^a \partial^\mu A_\mu^a \\ = & \left( 1 + \frac{1}{4\pi} g^2 C_V \right) [-i \operatorname{sgn}(k)] \int_x \epsilon^{\mu\nu\rho} \left( \frac{1}{2} A_\mu^a \partial_\nu A_\rho^a + \frac{1}{3!} A_\mu^a A_\nu^b A_\rho^c \right) + \frac{1}{4m} \left( 1 + \frac{9}{32\pi} g^2 C_V \right)^{-1} \\ & \times \int_x F^{\mu\nu a} F_{\mu\nu}^a - \Delta \left( \frac{1}{16\pi} g^2 C_V \int_x G_\mu^a A_\mu^a + \int_x L^a c^a \right) + \int_x B^a \partial^\mu A_\mu^a. \end{aligned} \quad (48)$$

<sup>1</sup>Rigorously speaking, the one-loop local effective action given here is not perfect, it should contain other  $(1/m)^n$  ( $n \geq 1$ ) dependent higher covariant derivative terms such as  $(1/m^{2n+1}) \int_x F_{\mu\nu}^a (D^2)^n F^{\mu\nu}$ ,  $(1/m) \Delta \int_x \epsilon^{\mu\nu\rho} G_\mu^a$ , and  $(1/m) \Delta \int_x (L^a c^a) (\bar{c}^b c^b)$ , etc. They also have correct mass dimension and ghost number. However, since in this section our aim is at the large  $m$  limit, we have put these  $1/m$  dependent terms out of consideration.

Finally the one-loop effective action of CS can be easily obtained by taking the large-mass limit  $m \rightarrow \infty$ . Obviously the wave-function renormalization constants are

$$Z_A = Z_B^{-1} = Z_G^{-1} = 1 - \frac{1}{16\pi} g^2 C_V, \quad Z_L = Z_C^{-1} = 1. \quad (49)$$

This result can be cast into  $k$ -shift form: i.e.,

$$k \rightarrow k + \text{sgn}(k) C_V. \quad (50)$$

## V. GAUGE FIELD SELF-ENERGY IN THREE-DIMENSIONAL QED

In the Appendix are given some formulas which are useful for computation in the study of any three-dimensional theory in coordinate space. Here, by the way, we would like to point out that, as an example, using these formulas we can get the one-loop self-energy part for the gauge field in three-dimensional massive QED an analytic expression whose integral form was given in Ref. [5];

$$\begin{aligned} \Pi_{ij}(p) = & -\frac{e^2}{8\pi} (\delta_{ij} p^2 - p_i p_j) \left[ \frac{2m}{p^2} + \left( \frac{1}{p} - \frac{4m^2}{p^3} \right) \arctan \frac{p}{2m} \right] \\ & - m e^2 \epsilon_{ijk} \frac{1}{2\pi p} \arctan \frac{p}{2m}, \end{aligned} \quad (51)$$

where the notation is the same as that in Ref. [5].

## VI. SUMMARY

We carried out the one-loop calculation of the topologically massive Yang-Mills theory and Chern-Simons theory in coordinate space using the method of differential renormalization. Our calculation shows that the method is very powerful and is especially suited for quantum field theories in three dimensions. The results we obtained on TMYM theory coincide with those of Ref. [9], which used the method of dimensional regularization. However, as was pointed out in Ref. [9], the calculus of dimensional regularization for a theory in three dimensions is subtle and perhaps even problematic, not least because of the need for a dimensional continuation of the antisymmetric tensor  $\epsilon_{\mu\nu\rho}$ ; it is not known to what extent the calculated renormalization of a field theory such as TMYM, whose property is closely tied to the dimension of space-time, could be an artifact of this continuation. In differential renormalization there is not such an ambiguity because one does not change the dimension of space-time, so there is no need for a continuation of the antisymmetric tensor. It is therefore reassuring that the two sets of results agree. For Chern-Simons field theory our result shows the shift  $k$  to  $k + \text{sgn}(k) C_V$ , which coincides with the case of scalar regulator of Ref. [19].

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## APPENDIX

### 1. Differential formulas

Defining

$$f^{(n)} \equiv \left( \frac{1}{r} \frac{d}{dr} \right)^n f(r), \quad f(r) = \frac{1 - e^{-mr}}{r}, \quad (A1)$$

and denoting  $\partial_1 = \partial_{i_1}$ ,  $x_1 = x_{i_1}$ , etc., we have

$$\begin{aligned} \partial_1 \partial_2 \cdots \partial_{2n} f = & (\delta_{12} \delta_{34} \cdots \delta_{2n-1, 2n} + \text{permutations}) f^{(n)} \\ & + (x_1 x_2 \delta_{34} \delta_{56} + \text{permutations}) f^{(n+1)} \\ & + (x_1 x_2 x_3 x_4 \delta_{56} \delta_{78} \cdots \delta_{2n-1, 2n} \\ & + \text{permutations}) f^{(n+2)} + \cdots \\ & + (x_1 x_2 \cdots x_{2n}) f^{(2n)}, \end{aligned} \quad (A2)$$

$$\begin{aligned} \partial_1 \partial_2 \cdots \partial_{2n+1} f = & (x_1 \delta_{23} \delta_{45} \cdots \delta_{2n, 2n+1} \\ & + \text{permutations}) f^{(n+1)} \\ & + (x_1 x_2 x_3 \delta_{45} \cdots \delta_{2n, 2n+1} \\ & + \text{permutations}) f^{(n+2)} + \cdots \\ & + (x_1 x_2 \cdots x_{2n+1}) f^{(2n+1)}. \end{aligned} \quad (A3)$$

Some examples are

$$\partial_i f = x_i f^{(1)},$$

$$\partial_i \partial_j f = \delta_{ij} f^{(1)} + x_i x_j f^{(2)},$$

$$\partial_i \partial_j \partial_k f = (x_i \delta_{jk} + x_j \delta_{ki} + x_k \delta_{ij}) f^{(2)} + x_i x_j x_k f^{(3)}. \quad (A4)$$

### 2. Fourier transforms of some functions

$$\int_x \frac{1}{x^2} e^{ip \cdot x} = \frac{2\pi^2}{|p|}, \quad (A5)$$

$$i \int_x \frac{e^{-nm x}}{x} e^{ip \cdot x} = \frac{4\pi}{n^2 m^2 + p^2},$$

$$\int_x \frac{e^{-nm x}}{x^2} e^{ip \cdot x} = \frac{4\pi}{p} \arctan \frac{p}{nm},$$



$$\int_x \text{Ei}(-nm x) e^{ip \cdot x} = -\frac{8\pi}{p^3} \left[ \frac{\pi}{4} - \frac{1}{2} \arctan \frac{mn}{p} - \frac{1}{4} \frac{mnp}{p^2 + m^2 n^2} \right].$$

### 3. Integrals over $R_\epsilon^3$

Recall that  $R_\epsilon^3$  is  $R^3$  excluding a small ball  $B_\epsilon$  of radius  $\epsilon$  about the origin.

$$\int_{R_\epsilon^3} e^{-ip \cdot x} \partial_\mu f(x) = -if(\epsilon) 4\pi \frac{p_\mu}{p} \frac{d}{dp} \left[ \frac{\sin(p\epsilon)}{p} \right] + ip_\mu \int_x e^{-ip \cdot x} f(x). \quad (\text{A6})$$

$$\int_{R_\epsilon^3} e^{-ip \cdot x} \partial_\mu \partial_\nu f(x) = 4\pi \frac{1}{\epsilon} \frac{d}{dx} f(x) \Big|_{x=\epsilon} \frac{\partial}{\partial p_\mu} \frac{\partial}{\partial p_\nu} \left[ \frac{\sin(p\epsilon)}{p} \right] + 4\pi f(\epsilon) \frac{\partial}{\partial p_\mu} \frac{\partial}{\partial p_\nu} \left[ \frac{\sin(p\epsilon)}{p} \right] - p_\mu p_\nu \int_x e^{-ip \cdot x} f(x). \quad (\text{A7})$$

$$\int_{R_\epsilon^3} e^{-ip \cdot x} \partial^2 \partial_\mu f(x) = -i(4\pi) \partial^2 f(x) \Big|_{x=\epsilon} \frac{\partial}{\partial p_\mu} \left[ \frac{\sin(p\epsilon)}{p} \right] + i4\pi p_\mu p_\alpha \epsilon f(\epsilon) \frac{\partial}{\partial p_\alpha} \left[ \frac{\sin(p\epsilon)}{p} \right] - i4\pi \epsilon p_\mu \frac{d}{dx} f(x) \Big|_{x=\epsilon} \frac{\sin(p\epsilon)}{p} - ip_\mu p^2 \int_x e^{-ip \cdot x} f(x). \quad (\text{A8})$$

$$\int_{R_\epsilon^3} e^{-ip \cdot x} \partial_\mu \partial_\nu \partial_\rho f(x) = i4\pi \delta_{\nu\rho} \frac{\partial}{\partial p_\mu} \frac{\sin(p\epsilon)}{p\epsilon} \left[ \frac{1}{x} \frac{d}{dx} f(x) \right]_{x=\epsilon} + i4\pi \frac{\partial^3}{\partial p_\mu \partial p_\nu \partial p_\rho} \frac{\sin(p\epsilon)}{p\epsilon} \frac{1}{x} \frac{d}{dx} \times \left[ \frac{1}{x} \frac{d}{dx} f(x) \right]_{x=\epsilon} - ip_\mu \int_x e^{-ip \cdot x} \partial_\nu \partial_\rho f(x). \quad (\text{A9})$$

### 4. Short-distance expansion

$$\frac{\partial}{\partial p_\mu} \left[ \frac{\sin(p\epsilon)}{p} \right] = p_\mu \left[ -\frac{\epsilon^3}{3} + \frac{p^2 \epsilon^5}{30} - \frac{p^4 \epsilon^7}{30} + \mathcal{O}(\epsilon) \right]. \quad (\text{A10})$$

$$\frac{\partial^2}{\partial p_\mu \partial p_\nu} \left[ \frac{\sin(p\epsilon)}{p} \right] = \delta_{\mu\nu} \left[ -\frac{\epsilon^3}{3} + \frac{p^2 \epsilon^5}{30} - \frac{p^4 \epsilon^7}{30} + \mathcal{O}(\epsilon) \right] - p_\mu p_\nu \left[ -\frac{\epsilon^5}{15} - \frac{p^2 \epsilon^7}{210} + \frac{p^4 \epsilon^9}{7560} + \mathcal{O}(\epsilon) \right]. \quad (\text{A11})$$

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